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LAG IN AIRCRAFT ALTITUDE MEASURING SYSTEMS

KIRK S. IRWIN
CHIEF, FLUID and FLIGHT
MECHANICS SECTION
FLIGHT RESEARCH BRANCH

January 1963

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LAG IN AIRCRAFT ALTITUDE MEASURING SYSTEMS

by **KIRK S. IRWIN**

CHIEF, FLUID and FLIGHT
MECHANICS SECTION
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**AIR FORCE FLIGHT TEST CENTER
EDWARDS AIR FORCE BASE, CALIFORNIA
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ABSTRACT

The nature of pressure lag in aircraft altitude measuring systems is discussed and several methods of lag correction are compared. Based upon an extensive experimental program, an improved method of lag correction is proposed. Using the lag correction procedure outlined in this report, indicated altitude can be corrected to true pressure altitude to an accuracy of 0.5 percent. Other instrumentation uncertainties prevent the attainment of greater accuracy.

The influence of the instrumentation system temperature and the pitot-static probe surface temperature on the pressure lag are shown through theoretical and experimental analysis. Instrumentation system temperature has a moderate effect on the lag. The probe surface temperature, however, has no discernible influence on the lag.

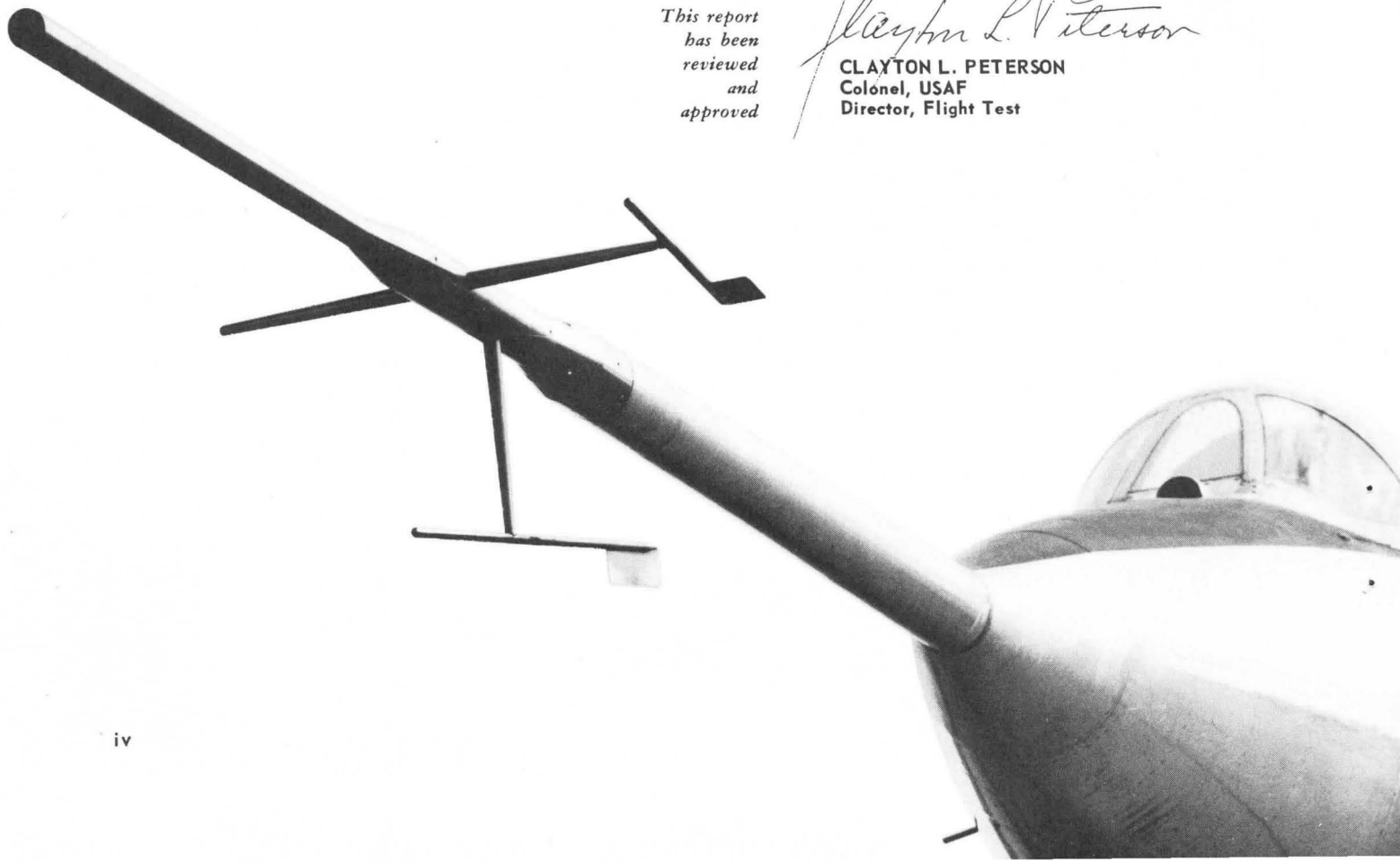
A system and procedure for ground lag checking on an aircraft static pressure system is presented in an appendix to the report.

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*This report
has been
reviewed
and
approved*

Clayton L. Peterson
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nomenclature

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
a _____	speed of sound _____	ft/sec
A _____	area _____	ft ²
D _____	diameter _____	ft
E _____	error _____	percent
f _____	function _____	
g _____	gravitational ac- celeration _____	ft/sec ²
H _____	pressure altitude _____	ft
L _____	length _____	ft
m _____	exponent in lag equation _____	
n _____	polytropic exponent _____	
P _____	pressure _____	lb/ft ²
t _____	time _____	sec
T _____	temperature _____	°K, °R
u _____	average velocity _____ across tube _____	ft/sec
V _____	volume (also valve designation in Appendix II) _____	ft ³
x _____	distance along tube _____	ft
β _____	lag parameter _____	sec
μ _____	viscosity _____	lb-sec/ft ²
ρ _____	density _____	slugs/ft ³
τ _____	acoustic time lag _____	sec

<u>Subscripts</u>	<u>Definition</u>
a _____	ambient
g _____	ground reference
i _____	indicated data corrected for instrument error
l _____	corrected for lag
o _____	reference condition
R _____	obtained from reference curve
t _____	true (corresponding to ambient or source condi- tions)
tb _____	tubing
SL _____	sea level



1 INTRODUCTION

Altitude is a parameter of prime importance in the flight test evaluation of aircraft performance. The accuracy of altitude measurement by means of a pressure sensing instrument, such as the aneroid barometer, is limited by the following possible sources of error:

1. Design of the static pressure source.
2. Flow field about the static pressure source.
3. Lag between pressure source and sensing instrument.
4. Mechanical characteristics of the sensing instrument.

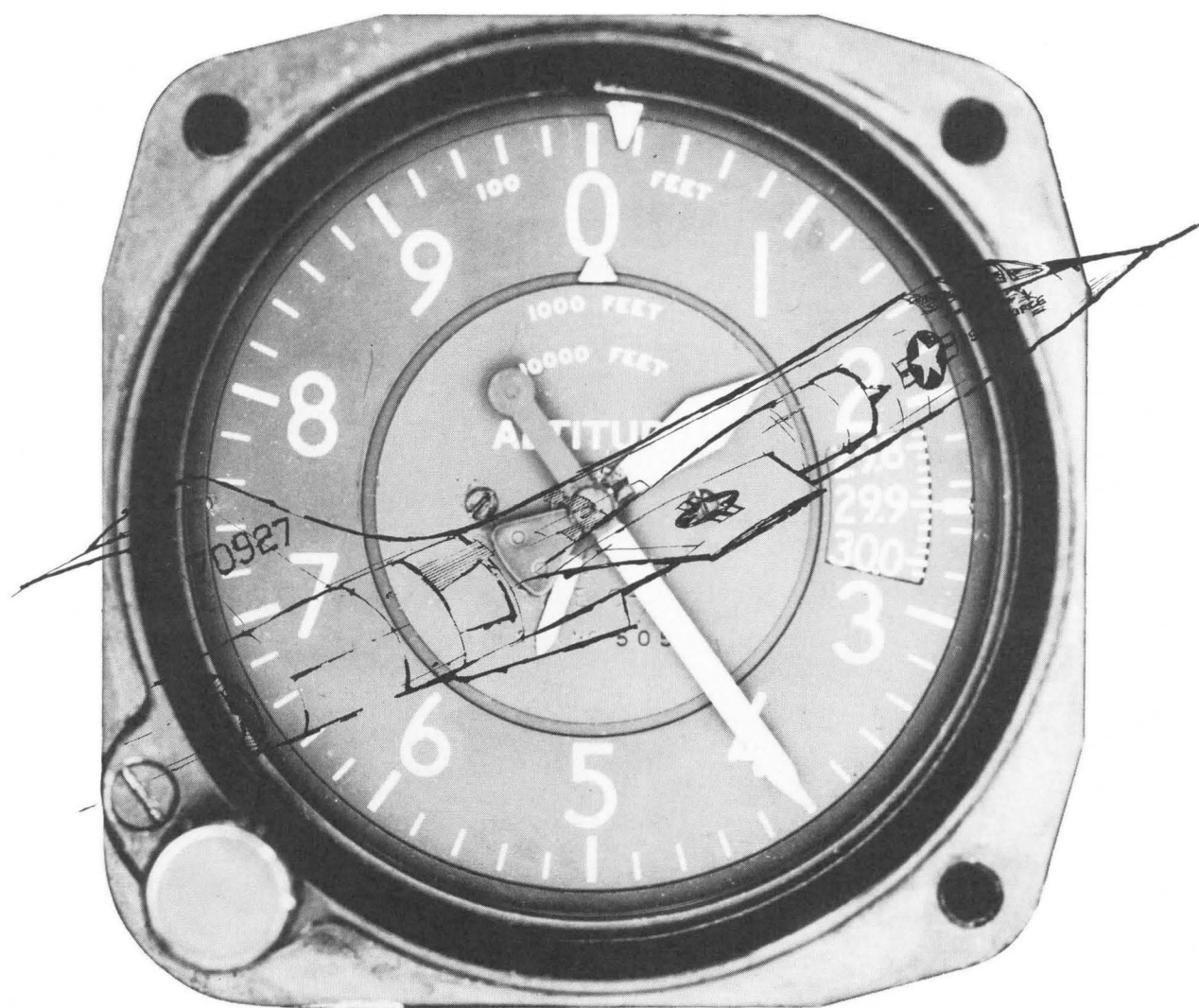
The third source of error, pressure lag, is the subject of this report.

The static pressure changes as the airplane changes altitude. Before the altitude measuring instrument, or altimeter, can correctly indicate the change in altitude, a finite amount of air must flow through the tubing between the static source and the altimeter. The pressure in the altimeter will lag behind the pressure at the static source by an amount related to the finite speed of pressure propagation through the system and the pressure drop due to flow through the tubing. The result, of course, is an erroneous indication or recording of altitude. Lag errors in excess of 1000 feet are not at all unrealistic for high rates of climb or descent.

In the early era of flight it was hardly important to measure altitude, let alone correct the measurement for lag. The performance of early aircraft was so limited that altimeter lag was not seriously studied until the mid - 1930's. The first important investigation of the subject was published in 1937 (Reference 1). But it was not until the advent of the turbojet aircraft near the end of World War II that interest in pressure lag in aircraft instrument systems really picked up. A number of studies have been made of the subject since 1945.

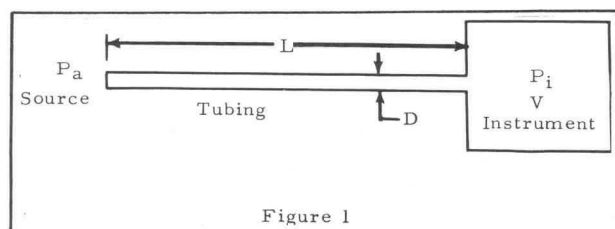
Many investigators have concentrated their efforts on the study of geometric factors which influence the lag. The general effects on lag of tubing length, tubing diameter, instrument volume, etc., are fairly well understood and will not be covered in detail in this report. The emphasis here will be to develop a reasonable means of correcting for the lag in an altitude measuring system.

In an attempt to arrive at an adequate method of correcting indicated altitude for lag, several theoretical approaches were studied and a series of laboratory tests were conducted. The study has shown that currently employed methods of lag correction are often inadequate for modern high performance aircraft. An improved method of lag correction will be described, and the accuracy of the method will be demonstrated.



2 ANALYSIS

The physical system to be analyzed in a study of lag in altitude measurement consists of a pressure source, commonly a nose or wing-mounted pitot-static boom, a length of tubing and plumbing fittings, and one or more instruments which indicate or record the variation of pressure. The detailed description of the flow throughout a static or total pressure system in an aircraft would be prohibitively complex. In one installation the line from the static source to the instrument panel may involve metal, rubber, and plastic tubing joined together with a variety of fittings. The tubing will not be straight but will be routed through available holes in bulkheads and around equipment. For the sake of simplicity in preliminary analysis, the system of tubing, fittings, and instruments will be considered to be a straight tube from the pressure source to a closed instrument of volume V .



The lag in the system may be expressed as the sum of the acoustic lag and the viscous lag. The acoustic lag occurs because of the finite time required for any pressure change at the source to be propagated to the instrument. The viscous lag is a result of fluid flow with an associated variation of pressure along the tube.

Acoustic lag is not difficult to compute. The time which will elapse between the instant a pressure disturbance occurs at the source and the instant it is first felt at the instrument will equal the length of the tube divided by the speed of sound in the tube.

$$\tau = \frac{L}{a} \quad (1)$$

The speed of sound in small diameter tubing may be approximated at $a = 1000$ feet per second. Then:

$$\tau = \frac{L}{1000} \text{ seconds} \quad (2)$$

For a 30 foot length of tubing, the acoustic lag would be only 0.03 second. As will be shown, the viscous lag will generally be at least one order of magnitude larger than the acoustic lag. For this reason, acoustic lag has been neglected in the subsequent analysis and experimental study. If a particular instrument installation does require consideration of acoustic lag, the correction given by equation (3) may be added to the viscous lag to obtain total lag.

$$\Delta P_{\text{acoustic}} = \tau \frac{dP}{dt} \quad (3)$$

The mathematical description of viscous flow in the system is not simple. It depends upon the nature of the forcing function (the variation of pressure at the source), the thermodynamic characteristics of the flow, and the geometry of the system. Within the past thirty years a number of theories have been proposed to describe pressure lag due to viscous flow. Essentially, four main methods of approach have been taken:

1. Analogy to current in a passive electrical circuit (Reference 2).
2. Analogy to displacement in a damped spring mass system (References 3, 4, and 5).
3. Analogy to mass flow through a sharp-edged orifice into a fixed volume (Reference 6).
4. Derivation of pressure difference based upon classical equations for conservation of mass and momentum (References 1, 7, 8, 9, and 10).

Each method results in a first or second order differential equation of varying degrees of complexity, depending upon the simplifying assumptions which were applied. An example of the more straightforward method of derivation based upon the equations for conservation of mass and momentum is presented in Appendix I.

Except for brief transient periods during initiation of some maneuvers the rate of change of density or pressure at the static source will be relatively steady. The second derivatives of density and pressure may be neglected for lag correction. When all terms containing $d^2 P/dt^2$ and $d^2 \rho/dt^2$ are omitted, the various theories can be summarized into the following general expression:

$$\Delta P = P_a - P_i = f_1 f_2 \frac{1}{n} \frac{1}{P_i} \left(\frac{dP_i}{dt} \right)^m \quad (4)$$

where:

f_1 = function of geometry
 f_2 = function of temperature

The various theories show disagreement on the forms of f_1 and f_2 and the value for n and m . Some treatments have resulted in $\left(\frac{1}{P_a} \frac{dP_a}{dt} \right)$ on the right side of equation 4. In each case, however, the equations can be transformed to the form shown in equation (4).

Table I illustrates the variations of f_1 , f_2 , n , and m for five of the references. Viscosity, μ , is a function of temperature alone; hence, it appears under the f_2 heading. When the flow is assumed to be isothermal, $n = 1$. When it is assumed to be isentropic, $n = 1.4$. When the only stipulation is that the flow be constrained by the polytropic relation, $P = \rho^n \cdot \text{const.}$, then n can have any value from 1.0 to 1.4.

n = exponent in the polytropic equation of state
 $(P = \rho^n \cdot \text{const.})$
 m = 1 or 2

TABLE I

Reference	f_1	f_2	n	m
Wildhack	$\frac{128L}{\pi D^4} (V)$	μ	1	1
Huston	$\frac{128L}{\pi D^4} (V + V_{tb})$	μ	1.4	1
Charnley	$\frac{128L}{\pi D^4} (V + \frac{V_{tb}}{2})$	μ	1	1
Smith	$\frac{128L}{\pi D^4} (V + \frac{V_{tb}}{2})$	μ	$1 \leq n \leq 1.4$	1
Vaughn	V^2/A^2	T/T_i^2	1	2

The flight test engineer generally does not have to concern himself with the forms of f_1 and f_2 ; nor does he need to know the value of n . Once an instrumentation system has been installed in an aircraft, the geometry is fixed so that f_1 is constant. The viscosity from the first four entries in Table I is the viscosity in the tubing and will be essentially constant if the internal temperature of the aircraft does not change too severely during flight. The term T/T_i from Vaughn's theory may also be considered constant. T is the temperature upstream of the smallest restriction in the system and T_i is the instrument temperature and will not vary enough to effect the results. Hence, in each case the temperature expression f_2 may be assumed to be a constant.

Whether the heat transfer characteristics of the system are such that the flow is isentropic, isothermal, or somewhere in between, the value of n is constant.

Joining the constant terms into one parameter, β , equation (4) can be written:

$$\Delta P = \beta \frac{P_{SL}}{P_i} \left(\frac{dP_i}{dt} \right)^m \quad (5)$$

where:

$$\beta = f_1 f_2 \frac{1}{nP_{SL}} \quad (6)$$

The fundamental assumptions employed in the development of equation (5) are:

1. Laminar flow
2. Negligible second order derivatives $\left(\frac{d^2 P}{dt^2} \text{ and } \frac{d^2 \rho}{dt^2} \right)$

The critical Reynold's number for transition from laminar to turbulent flow in tubes is about 2000. The flow of air in the static pressure system will never exceed the critical Reynold's number under flight conditions except possibly near fittings, orifices, and sharp bends in the tubing. The assumption of negligible second derivative terms was briefly discussed previously. It implies that the inertial forces on the fluid are much smaller than the viscous forces, a fact that is substantiated by the low Reynold's number of the flow.

The parameter β depends upon the geometry and the thermodynamic characteristics of the system. It is not dependent upon the nature of the variation of the source pressure provided that the assumptions of laminar flow and negligible second order terms are not violated. Determination of β by step function pressure changes at the source would violate the restrictions.

The problem of correcting for lag in altitude measuring systems has reduced to the problem of determining m and β . The exponent, m , may be either one or two. Interpretation of this analysis to permit fractional values of m would be erroneous. Any such determination of m would be purely empirical and the results could not be generalized to any arbitrary system.

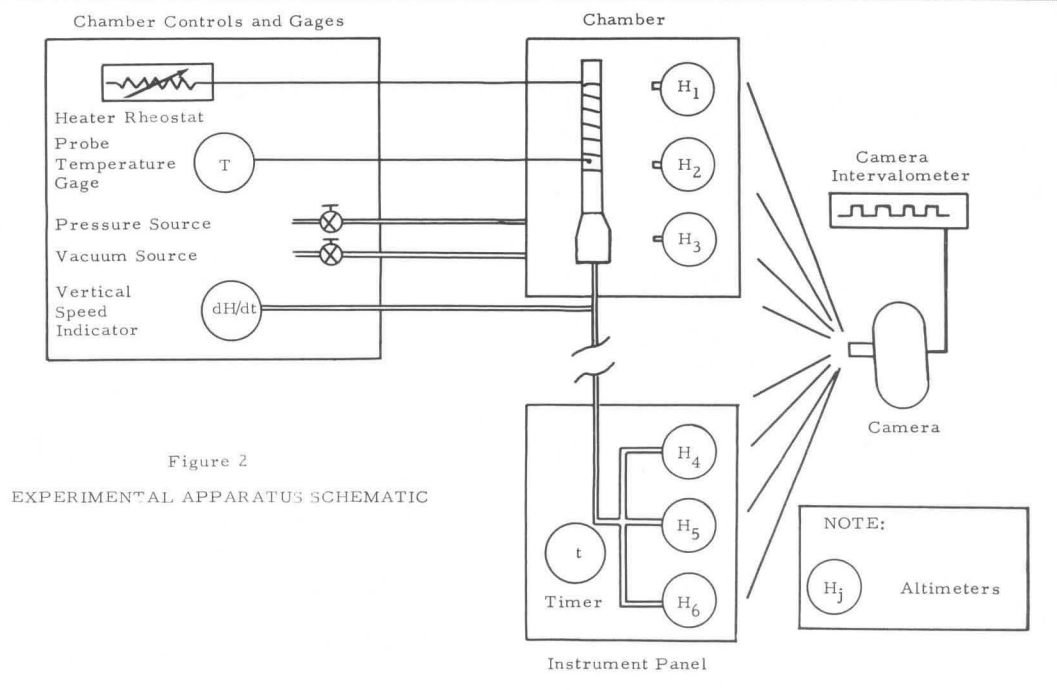
Theory predicts that β should be a constant. Experimental procedures were employed to determine if such is, in reality, the case. The description of the experiments performed and the results follow.

3 EXPERIMENT

The experimental apparatus shown schematically in Figure 2 was used to simulate a realistic aircraft static pressure system. The 'ambient' or source pressure of the simulated aircraft is the chamber pressure which is controlled by the pressure and vacuum valves. To simulate a climb, the chamber pressure was changed by manipulation of the vacuum valve. To simulate a descent, the chamber was initially set at a low pressure corresponding to an altitude of about 80,000 feet. Then the pressure source valve was used to control the descent. The pressure changes were controlled manually. Since altitude lag was the parameter of prime interest, the system was designed for direct control of altitude rather than pressure. The desired rate of change of altitude was maintained by visually monitoring the "vertical speed" (dH/dt) indicator.

The aircraft static pressure instrumentation system was represented by an actual pitot-static probe located in the chamber with a length of tubing and fittings passing from the probe through the chamber wall to a set of altimeters mounted on a photopanel. The tubing was coiled (large bend radius) so that the instrument panel could be adjacent to the chamber. A timer was mounted on the instrument panel.

Three altimeters were mounted near a window in the chamber. They vented directly into the chamber; hence, their indications were essentially free of all lag other than the mechanical lag of the instrument's moving parts. Three altimeters in the chamber and three on the instrument panel were used so that average values could be obtained to counteract the possible distortion of results due to the peculiarities of one particular altimeter. The use of several altimeters also permitted a limited evaluation of other potential errors in the altitude measuring system.



There has been some speculation that the viscosity increase near the static ports due to aerodynamic heating of a pitot-static probe during high speed flight might increase the pressure lag. To test for such an effect, a heater coil was wrapped around the probe and a thermocouple was placed near the static ports. The probe temperature was controlled by varying the current through the heater coils.

All data was recorded by a 35mm motion picture camera.

For most test runs, the rate of change of altitude was kept reasonably constant. Both climbs and descents were simulated at rates between about 5,000 and 35,000 feet per minute. Due to inaccuracy of the vertical speed indicator, the rates could not be held perfectly constant, especially at high rates. Figure 3 shows the variation of dH/dt with altitude for test runs 1 through 12. The data has been smoothed by the method of least squares. In each case the dH/dt instrument indicated a constant rate but the true rate varied as shown.

The corresponding lag as a function of altitude for the same test runs is shown in Figure 4. The curves in Figure 4 have been faired by hand for simplicity of presentation.

Note that at high altitudes and high rates of climb and descent the indicated altitude lagged the chamber or true altitude by thousands of feet. Such high values of altitude lag are not unique to this laboratory simulation. They are characteristic of many aircraft instrument systems.

The real test of the validity of the prediction of, or correction for, pressure lag by means of equation (5) lies in the determination of β and m . Is β a constant? Does m equal one or two?

Figure 3
SIMULATED VERTICAL SPEED VS ALTITUDE
FOR EXPERIMENTAL RUNS 1-12

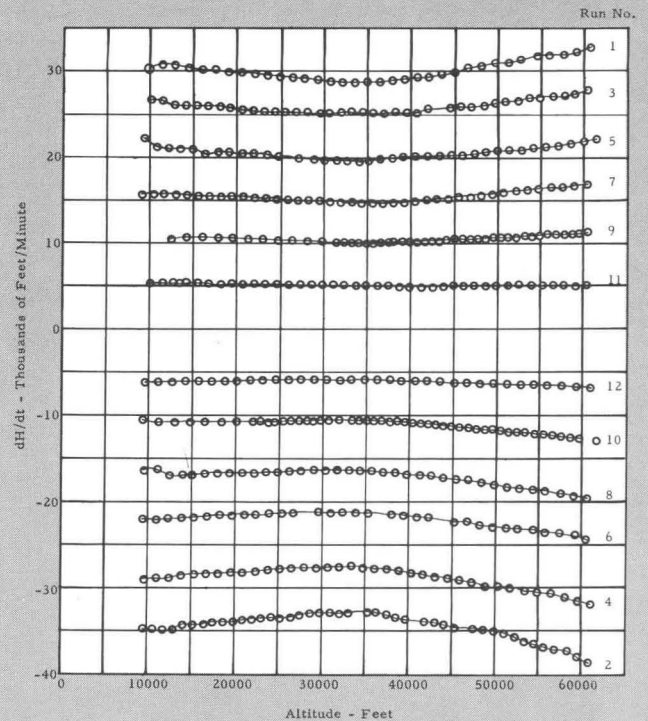
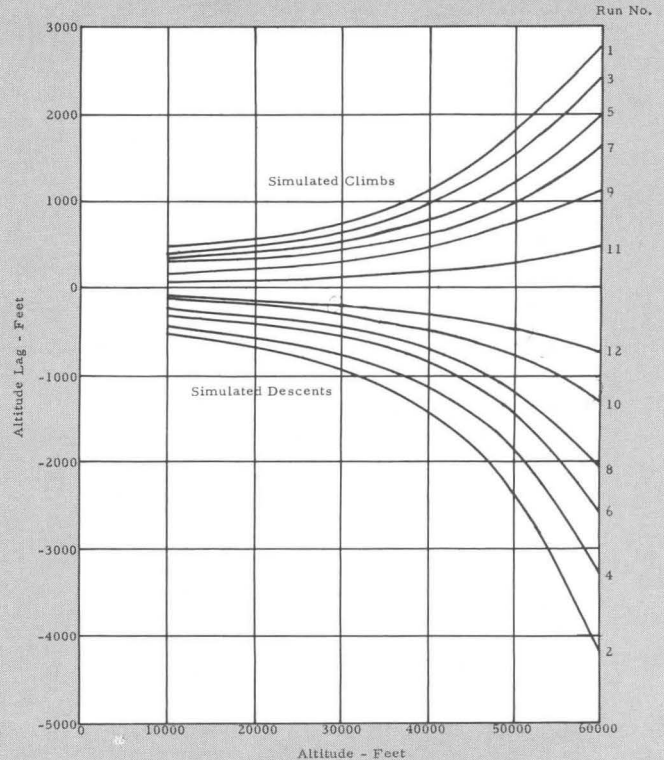


Figure 4
LAG AS A FUNCTION OF ALTITUDE
FOR EXPERIMENTAL RUNS 1-12



Data which has been reduced to determine β with $m = 1$ for test runs 1 through 12, is plotted in Figure 5. The data has been smoothed by the method of least squares. The vertical bars represent the scatter of the original data. The curves of Figure 5 indicate quite certainly that β is not a constant. There is an obvious dependence upon altitude, and comparison of the curves reveals a dependence on rate of change of altitude as well. To show the latter a cross-plot of the data at 60,000 feet is shown in Figure 6. Some data from another set of test runs with longer tubing between the chamber and the indicating instruments is also included for reference. The dependence of β on dH/dt is strongest at high altitudes (above 40,000 feet).

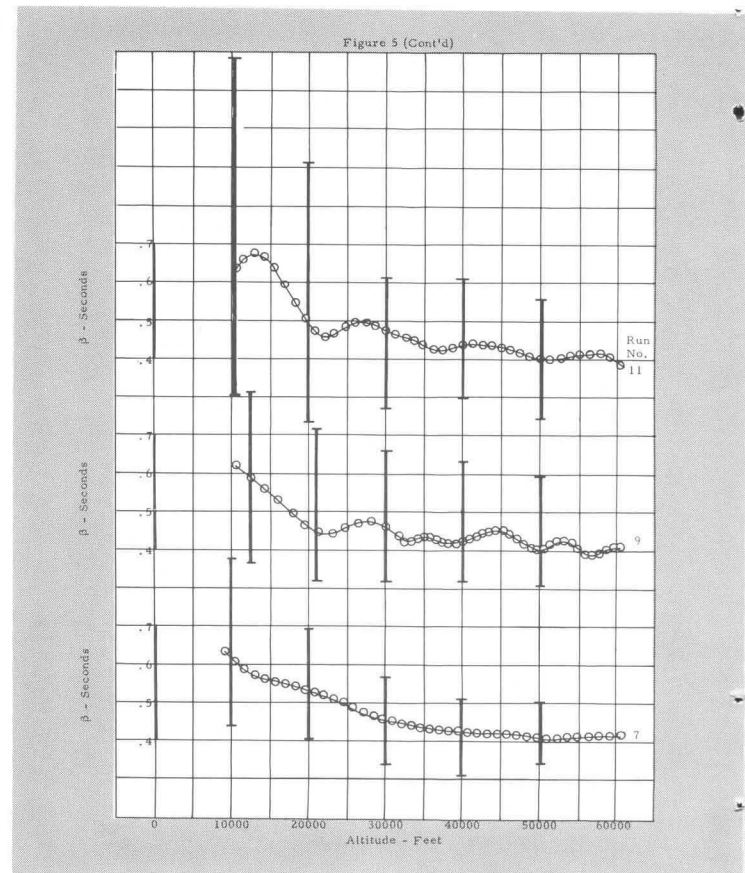
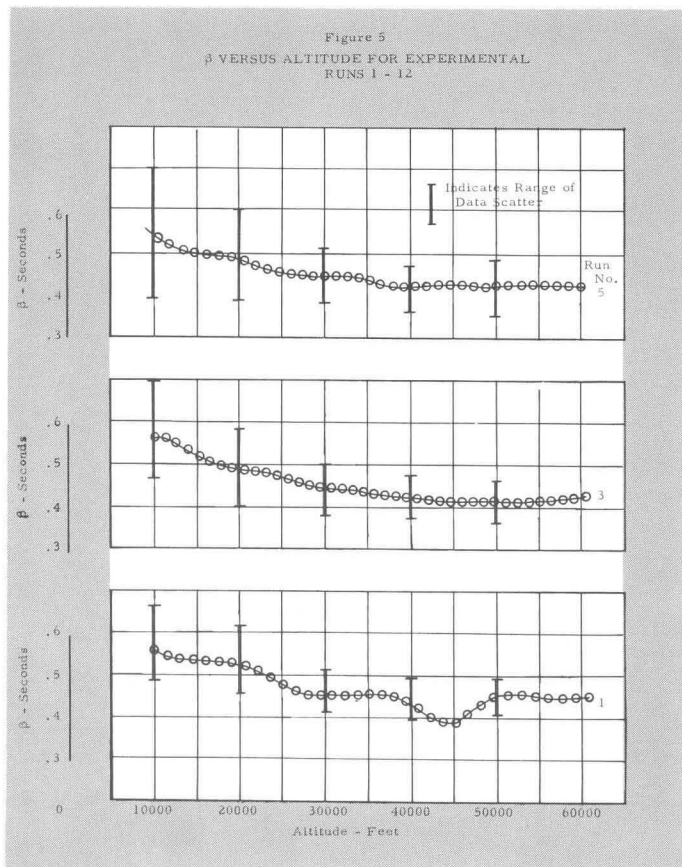
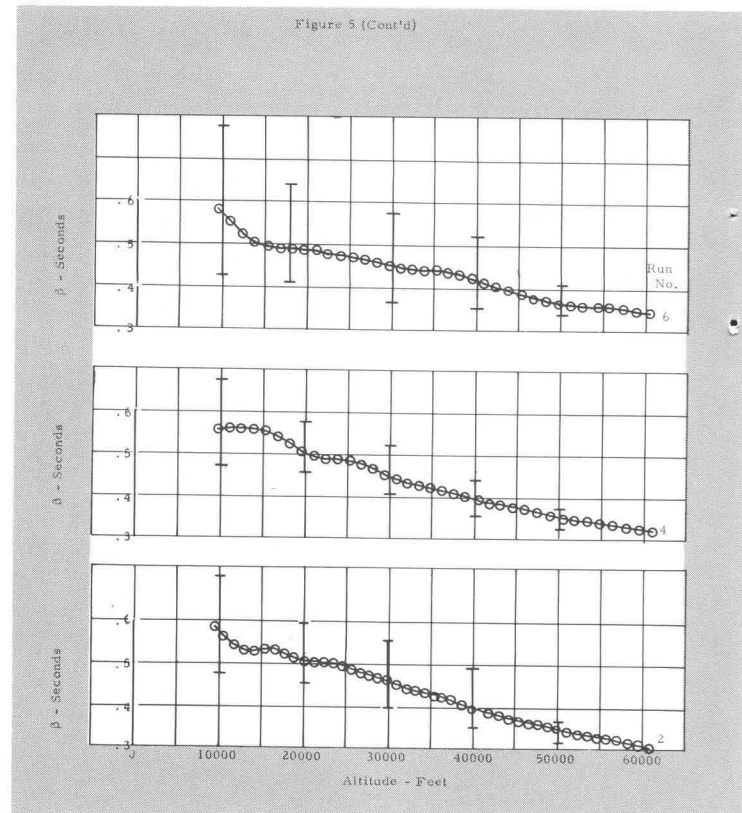


Figure 5 (Cont'd)

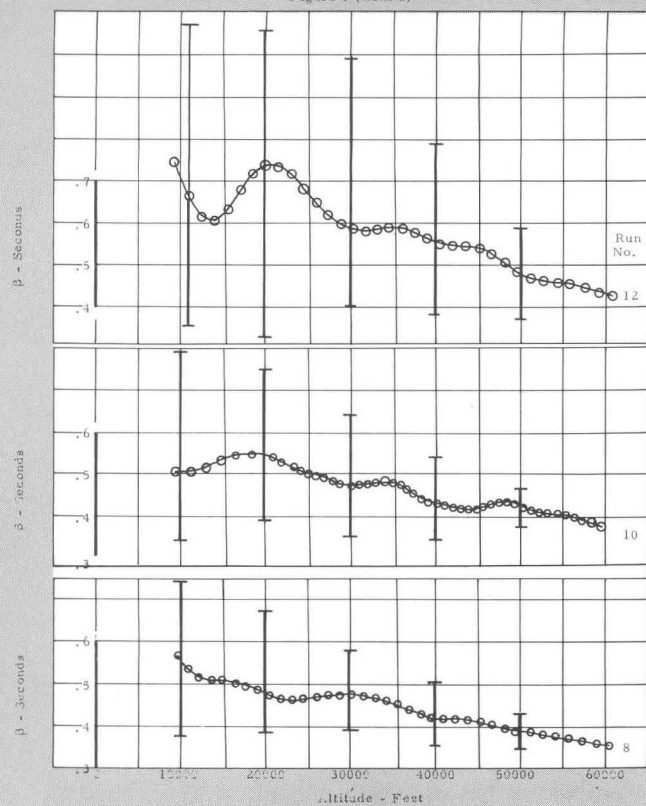
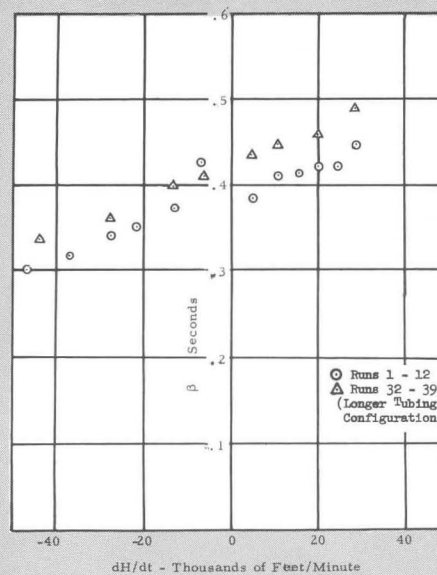


Figure 6
VARIATION OF β WITH dH/dt AT 60,000 FEET

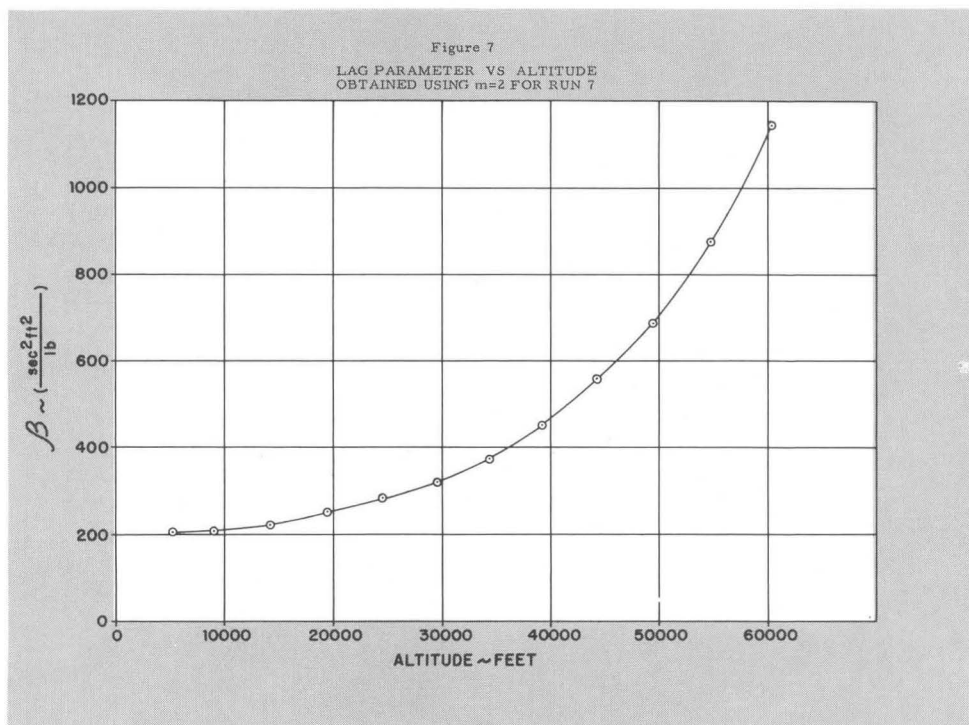


When the data was reduced with the exponent, m , taken to be the integer two, the parameter β exhibited more variation with altitude and rate of change of altitude than when $m = 1$ was used. Figure 7 shows β as a function of altitude for run number 7. The percentage change in β as a function of altitude is larger for $m = 2$ than for $m = 1$. The dependence on dH/dt of β for $m = 2$ was briefly analyzed and turned out to be more pronounced than for $m = 1$. Based upon these findings no further attempt was made to correct for lag in the system using $m = 2$. No attempt was made to find a fractional value of m which would result in values of β more nearly constant with H or dH/dt . For a given instrument system it might be possible to so determine such an exponent, but doing so would not necessarily mean that the same value would apply to another system.

The parameter β is a function of the system geometry (f_1), temperature (f_2), and the nature of the flow process (n). For a fixed instrument system the geometry of the installation will not change except for a negligible volume change of the aneroid in the altimeter. The polytropic exponent, n , will be either constant or so nearly constant that its variations will be negligible. But what about the temperature function f_2 ? Could a variation of f_2 be sufficient to explain the variation of β with H and dH/dt ?

Having essentially eliminated Vaughn's theory (see Table I) from further consideration based upon data showing $m = 2$ to result in greater departure from theory than methods suggesting $m = 1$, the only form for f_2 remaining is $f_2 = \mu$.

Viscosity μ is directly related to temperature and can be described quite adequately by the Sutherland viscosity formula

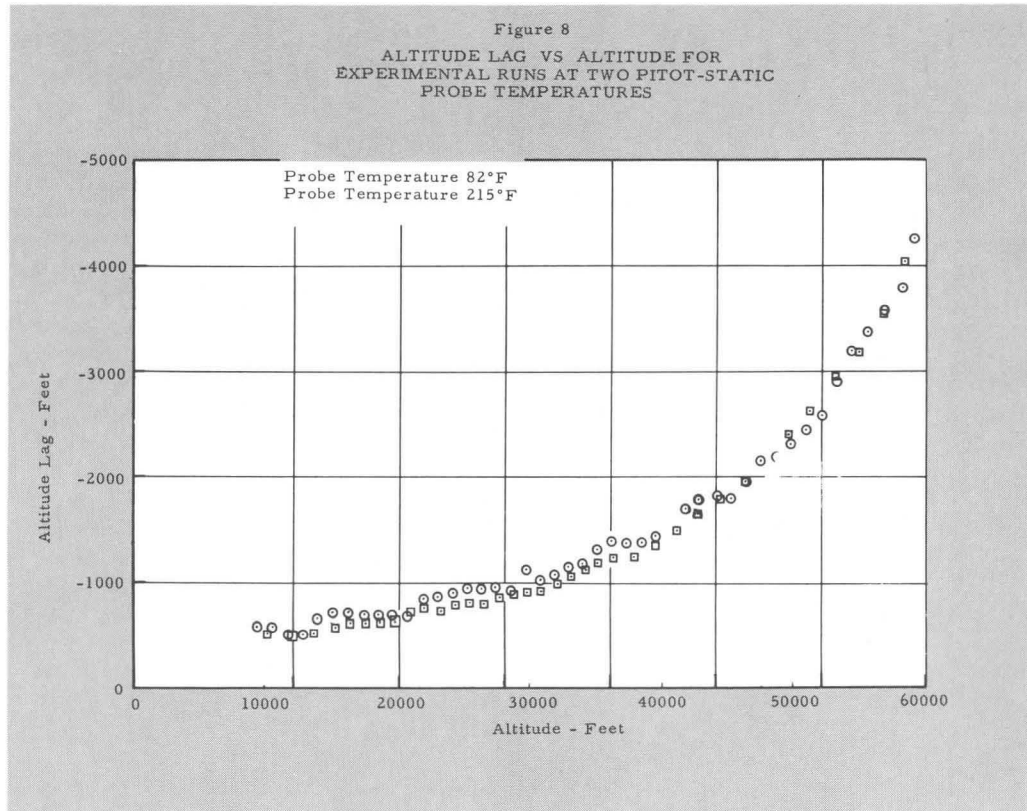


$$\frac{\mu}{\mu_0} = \left(\frac{T}{T_0} \right)^{3/2} \left(\frac{T_0 + S}{T + S} \right) \quad (7)$$

where μ_0 and T_0 are the viscosity and temperature for some reference condition and S is the Sutherland constant. S is 198.6 for temperature in degrees Rankine or 110.4 for temperature in degrees Kelvin. The temperature to be used in calculating viscosity is the temperature of the air in the tubing. A mean temperature may be used. As will be shown subsequently, the effect of errors in determining this temperature is generally not significant when applying a correction for lag.

For the laboratory runs the temperature of the chamber, the interconnecting tubing, and the indicating instruments was uniform and constant. Variations never exceeded 2.0 degrees F. It can then be concluded that the variability of β in the test runs cannot be attributed to its temperature dependence.

As pointed out earlier, there has been speculation that lag may be influenced to some measurable extent by the temperature of the sensor. The theory is that the high temperature of the pitot-static probe at high supersonic velocities could increase the viscosity of the air at the static ports enough to noticeably increase the lag. Several test runs were accomplished with the probe heated to about 200 degrees F by the heater coil shown in Figure 2. The data shown in Figure 8 illustrates that for a probe temperature of 215 degrees F no measurable increase in the lag took place. The graphs of dH/dt versus H for both runs were essentially identical. The only difference between the runs was the difference in the temperature of the probe; yet there was no effect on the plot of ΔH versus H . One can conclude then that the surface temperature of the pitot-static probe is not an important factor in determining the system lag.



4 CORRECTION FOR LAG

The basic problem at hand is to develop an adequate method of correcting for the lag that occurs in an altitude measuring system. The data presented in the preceding section has illustrated that none of the theoretical approaches correctly describes the existing flow process. Theoretically the term β should be a constant. But experimentally β depended upon both altitude and rate of change of altitude. It will now be shown that an adequate lag correction can be made using equation (5), but assuming that β can be a variable. For $m = 1$ equation (5) becomes

$$\Delta P = \beta \frac{PSL}{P_i} \frac{dP_i}{dt} \quad (8)$$

A pressure difference can be related directly to an altitude difference through the application of the differential relationship

$$dP = \rho g dH \quad (9)$$

which, for small differences, can be written

$$\Delta P = \rho g \Delta H \quad (10)$$

Using the relationships of equations (9) and (10), equation (8) becomes

$$\Delta H = \beta \frac{PSL}{P_i} \frac{dH_i}{dt} \quad (11)$$

Equation (11) is the basic expression for lag in an altimeter system. It only differs from expressions in many of the references in that β is a variable rather than a constant. If β is known as a function of indicated altitude or rate of change of indicated altitude, then the altitude lag, ΔH , can be determined directly from the test data with no difficulty at all. The obvious problem is that of determining β .

The most straight forward approach would be to conduct a series of experiments on the given instrument system to determine β at all altitudes and rates of change of altitude which might reasonably be expected in flight. For maximum accuracy in correcting for lag such a procedure would be necessary. However, considering the number of unknown errors possible in altitude

measurement a less complicated function of β can be determined. If a certain amount of error can be tolerated the number of curves required can be reduced. Rather than having a family of curves for β versus H at various values of dH/dt , it would be desirable to have just one or two curves.

Assume for the moment that a curve for β as a function of indicated altitude for one particular dH_i/dt has been determined by some method (the method will be described later). What error is introduced when using this curve to correct for lag at some other indicated rate of change of altitude? The error, as a percentage of true pressure altitude, would be

$$E = 100 \frac{\Delta H_{\beta_t} - \Delta H_{\beta_R}}{H_t} \quad (12)$$

In equation (12), ΔH_{β_t} is the true lag correction obtained by using a reference β versus H curve such as the sample in Figure 9.

$$\Delta H_{\beta_t} = \beta_t \frac{PSL}{P_i} \frac{dH_i}{dt} \quad (13)$$

$$\Delta H_{\beta_R} = \beta_R \frac{PSL}{P_i} \frac{dH_i}{dt} \quad (14)$$

If the expressions for ΔH_{β_t} and ΔH_{β_R} from equations (13) and (14) are substituted into equation (12), one obtains

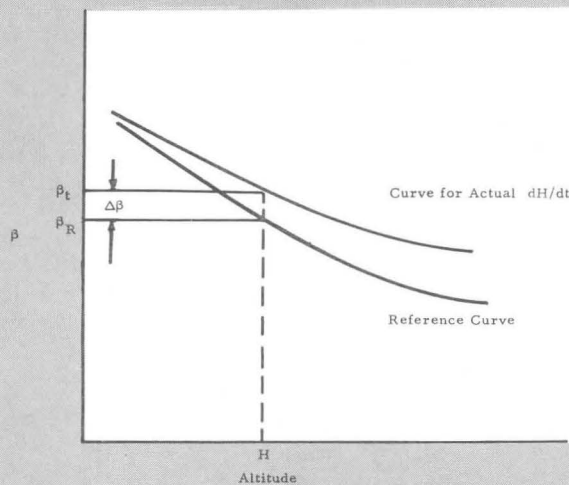
$$E = 100 \frac{(\beta_t - \beta_R) \frac{PSL}{P_i} \frac{dH_i}{dt}}{H_t} (\%) \quad (15)$$

$$E = 100 \frac{\Delta \beta \frac{PSL}{P_i} \frac{dH_i}{dt}}{H_t} (\%) \quad (16)$$

For the purpose of this error calculation, the small difference between P_i and P_a is negligible and P_i can be replaced by P_a .

$$E = 100 \Delta \beta \frac{PSL}{P_a H_t} \frac{dH_i}{dt} (\%) \quad (17)$$

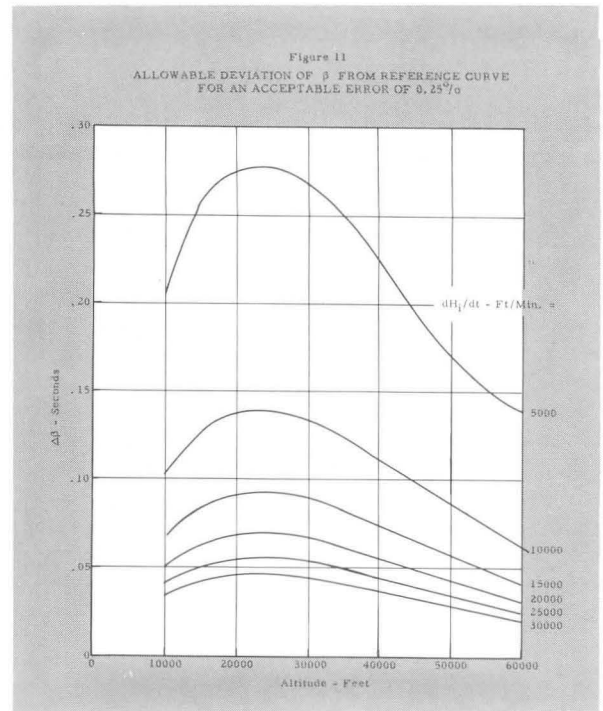
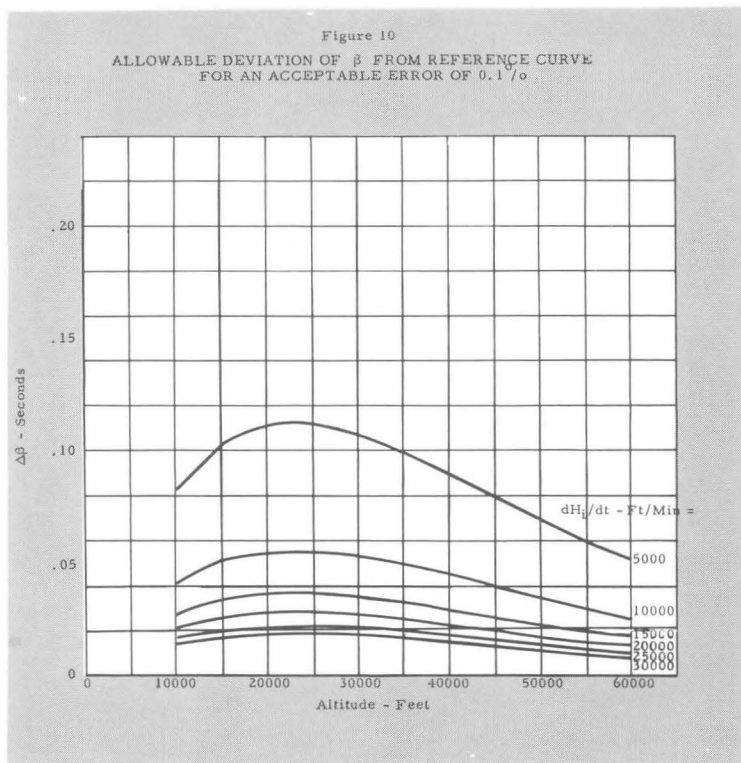
Figure 9
ILLUSTRATION OF REFERENCE
 β VS H CURVE



The ambient pressure, P_a , and the true altitude, H_t , are functionally related. For the standard atmosphere the equations relating the two terms are known. Equation (17) can be rearranged to yield the permissible deviation of β as a function of pressure altitude, rate of change of altitude, and the allowable error, E .

$$\Delta\beta = \frac{E}{100} \frac{H_t P_a / P_{SL}}{dH_i / dt} \quad (18)$$

Figures 10 and 11 show the graphs of equation (18) for allowable errors of 0.1 percent and 0.25 percent, respectively. Examination of the two figures reveals that the correct determination of β is most critical at high altitudes and high rates of change of altitude. At low values of dH_i/dt rather large variations may exist between the actual value of β and the value used in the lag correction for a given allowable error.



The statement has been made that it would be desirable to limit the number of β versus H curves necessary for lag correction to just one or two. Further, it has been shown that an accurate determination of β is most important for high values of dH/dt . Therefore, to assure reasonable accuracy of lag correction without undue complication of system lag checking, a plot of β versus H for one value of dH/dt near the maximum rate of interest will be sufficient. For example, if the airplane being tested is a fighter with a maximum rate of climb of 30,000 feet per minute, then β could be determined for 20,000 feet per minute and the resultant β versus H curve would offer acceptable accuracy even for low rates near 5,000 feet per minute.

Another factor to note is that the value of β at given values of H and dH_i/dt differs for climbs and descents. From Figure 6 it can be seen that the variation of β between a climb at 20,000 feet per minute and a descent at the same rate can be about 20 percent. Therefore, it is suggested that two curves for β versus H be obtained, one for a climb and one for a descent. For the above example, curves of β versus H for

both climb and descent at 20,000 feet per minute would be desirable. A system for lag checking an aircraft instrument installation is described in Appendix II. Using this system, data for β as a function of altitude can be obtained for any value of dH/dt .

A discussion of the selection of β versus H curves for lag correction must include a brief comment on data scatter. The scatter for the laboratory tests has been illustrated by the vertical bars on the curves of Figure 5. The data and equations from which β is computed generate considerable scatter which appears to be inversely related to dH/dt . The scatter occurs primarily because of the mechanical behavior of the instruments. A close analysis of the film records revealed some rather jerky and oscillatory behavior of the instruments even though they were all being gently vibrated to minimize sticking. The better quality of the data at higher values of dH/dt is additional justification for using β versus H curves for rates of change of altitude near maximum values in the lag correction.

If the average temperature of the instrumentation system tubing during flight differs appreciably from the temperature during the ground lag check when data is taken to find β , a temperature correction for β may be necessary. With $f_2 = \mu$ equation (6) becomes

$$\beta = f_1 \mu \frac{1}{nP_{SL}} \quad (19)$$

Since f_1 and n do not vary with temperature, the ratio of the value of β for the airborne case to the value found from ground check, β_g is

$$\frac{\beta}{\beta_g} = \frac{\mu}{\mu_g} \quad (20)$$

Then employing the relationship of equation (7)

$$\frac{\beta}{\beta_g} = \left(\frac{T}{T_g} \right)^{3/2} \left(\frac{T_g + S}{T + S} \right) \quad (21)$$

where T (without subscript) is the estimated mean temperature of the air-

craft instrument system in flight and T_g is the temperature of the system during ground lag check. S is 198.6 for temperature in degrees Rankine and 110.4 for temperature in degrees Kelvin.

Since the "average" temperature of the system is a rather ambiguous term, a look at the accuracy required in specifying this temperature would be helpful. The easiest way to do that is to note the deviations in corrected altitude due to the use of different temperatures in determining β . For purposes of this analysis, assume that the correct system temperature is 100 degrees F. The point now is to observe how much corrected altitude would be in error if temperatures other than 100 degrees F were used in the lag correction determination. The deviation or error is

$$E = 100 \left(1 - \frac{H(T=x^\circ)}{H(T=100^\circ)} \right) \% \quad (22)$$

To obtain $H(T=100^\circ)$, the parameter β is corrected by equation (21) for $T = 100^\circ\text{F}$. The term $H(T=x^\circ)$ is the lag corrected altitude with the same values of H_i , P_i , and dH/dt , but with β adjusted for an arbitrary temperature.

Figure 12 shows typical deviations in corrected altitude as calculated from equation (22). Experimental values of β_g were adjusted for temperatures between -50 degrees and +250 degrees F and applied to the lag corrections.

At low dH/dt , the data analyst has great latitude in selecting the temperature at which to compute β . In fact, within reasonable bounds, the temperature used has little effect on the correction. At higher values of dH/dt , and particularly at very low or very high altitudes, the determination of temperature becomes somewhat more important. Due to the inherent difficulty in specifying the exact role of temperature in the flow process it is difficult to more adequately define the average temperature. If large temperature variations exist along the tubing from the static source to the instruments, the selection of an appropriate temperature from which to compute β will be difficult. The ultimate choice must

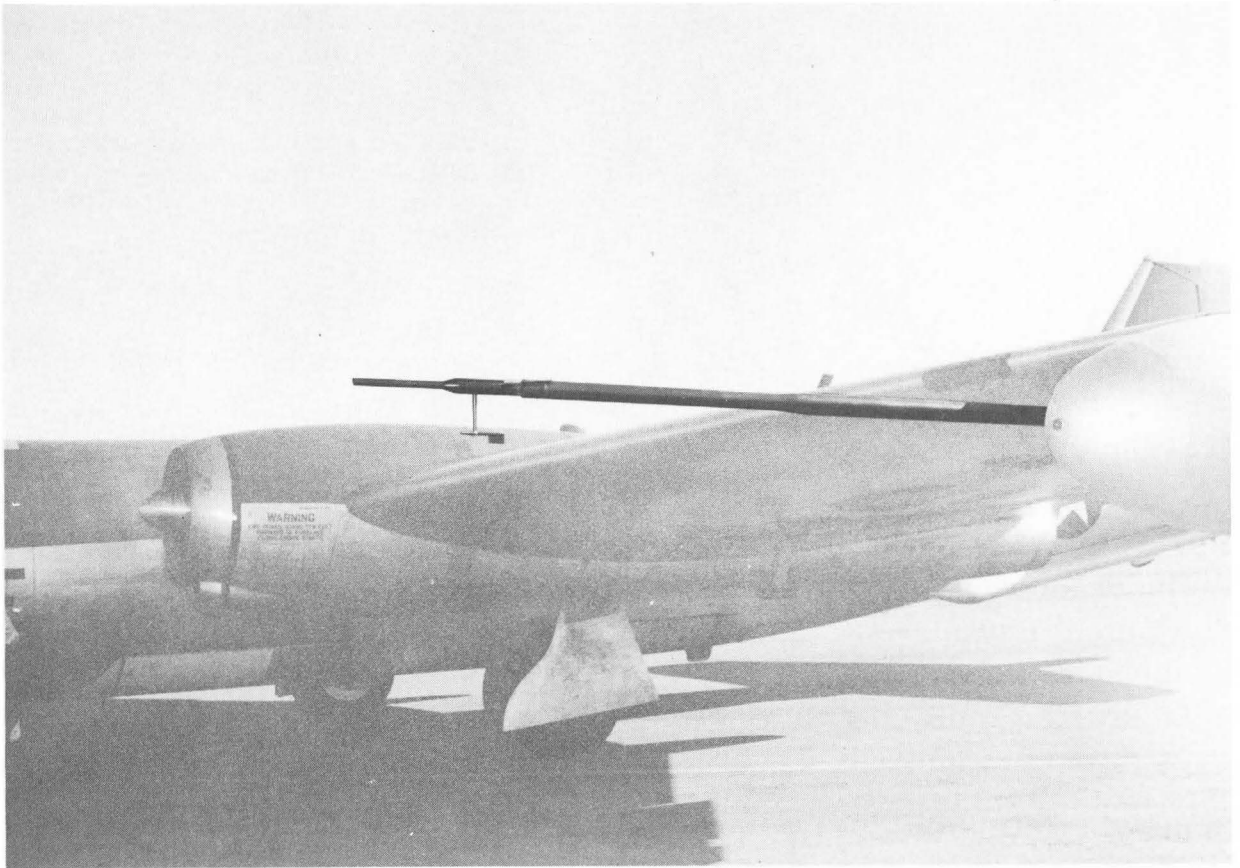
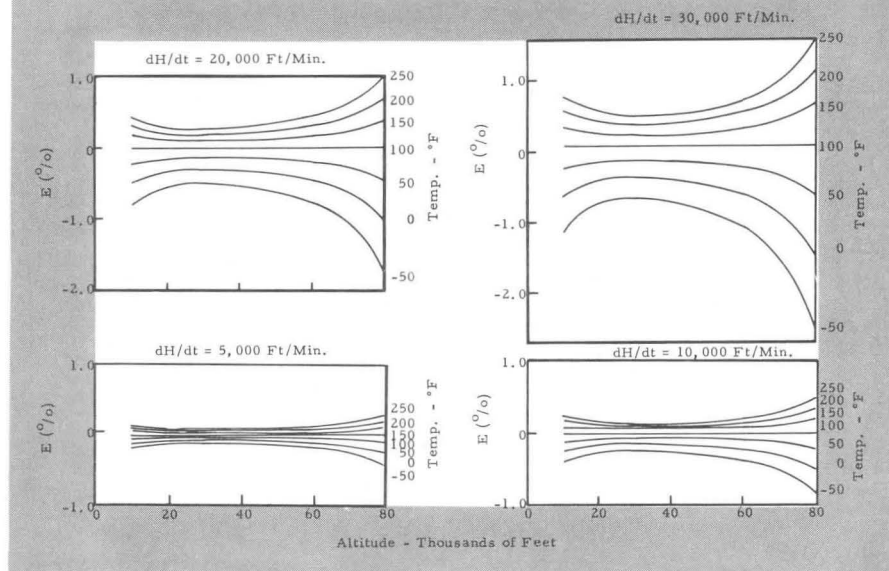


Figure 12
INFLUENCE OF INSTRUMENT SYSTEM TEMPERATURE
ON ACCURACY OF LAG CORRECTED ALTITUDE



rest with the data analyst and his knowledge of the instrumentation system.

The real test of the suggested lag correction method employing variable rather than constant β lies in its application to the correction of data which has not been ideally controlled (constant dH/dt , etc). Test runs were made wherein the altitude was varied in a somewhat random (yet reasonable) manner. The rates of change of altitude were allowed to vary to simulate a maneuvering aircraft. No serious attempt was made to simulate any particular maneuver, and perhaps the altitude time history is unrealistic, but it will provide a valid test of the correction method. The instrument system used was the same as that shown in Figure 2, but the tubing between the chamber and the indicating instrument panel was longer than the tubing used to obtain the data previously presented in this report.

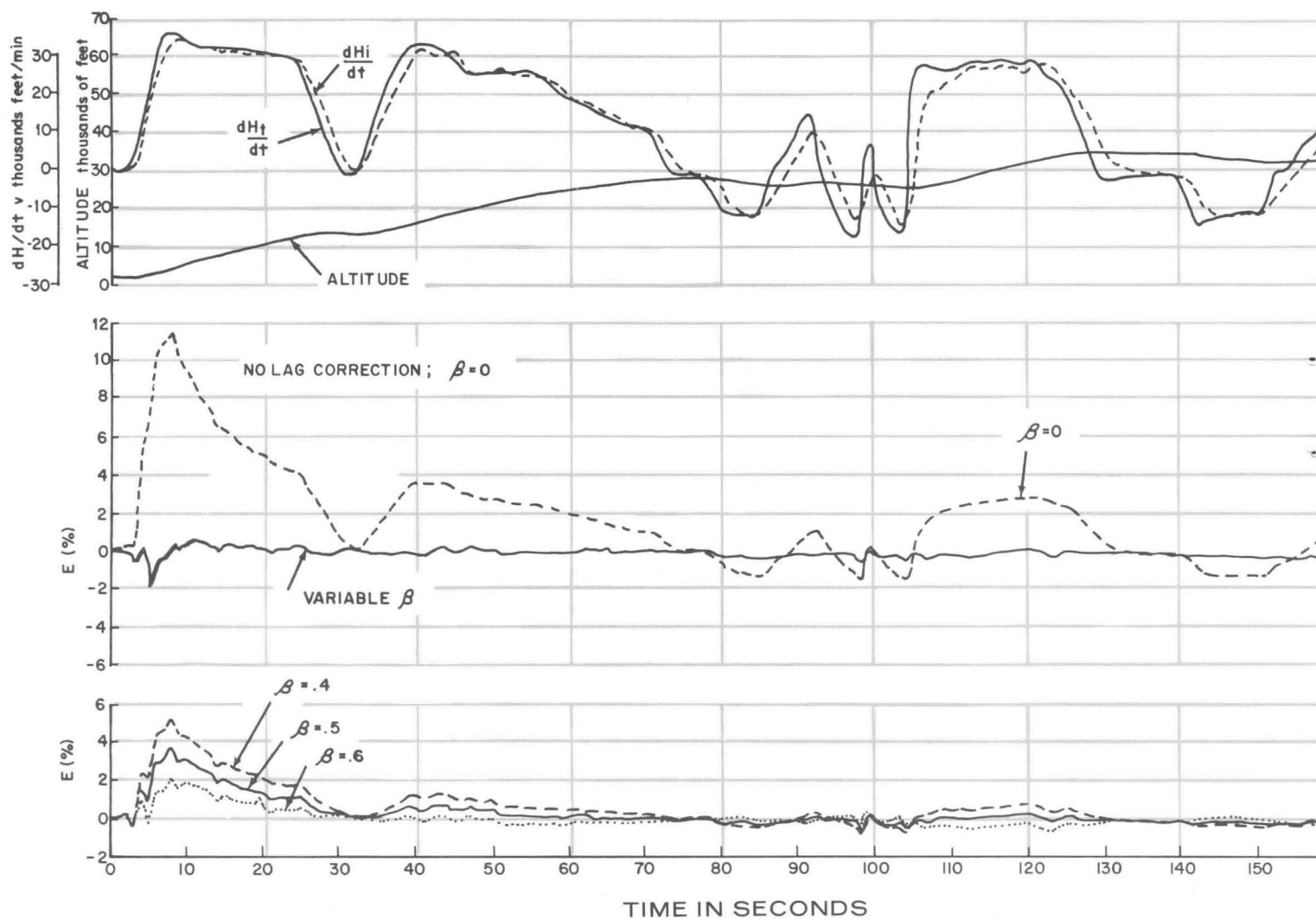
The indicated altitude data, H_i , was corrected for lag using various constant values of β . Then the data was corrected with variable β taken from curves similar to Figure 5. The time histories of the net altitude error for each correction technique is presented in Figure 13 along with the time histories of altitude, rate of change of true pressure altitude, and rate of change of indicated altitude. The net error is computed from

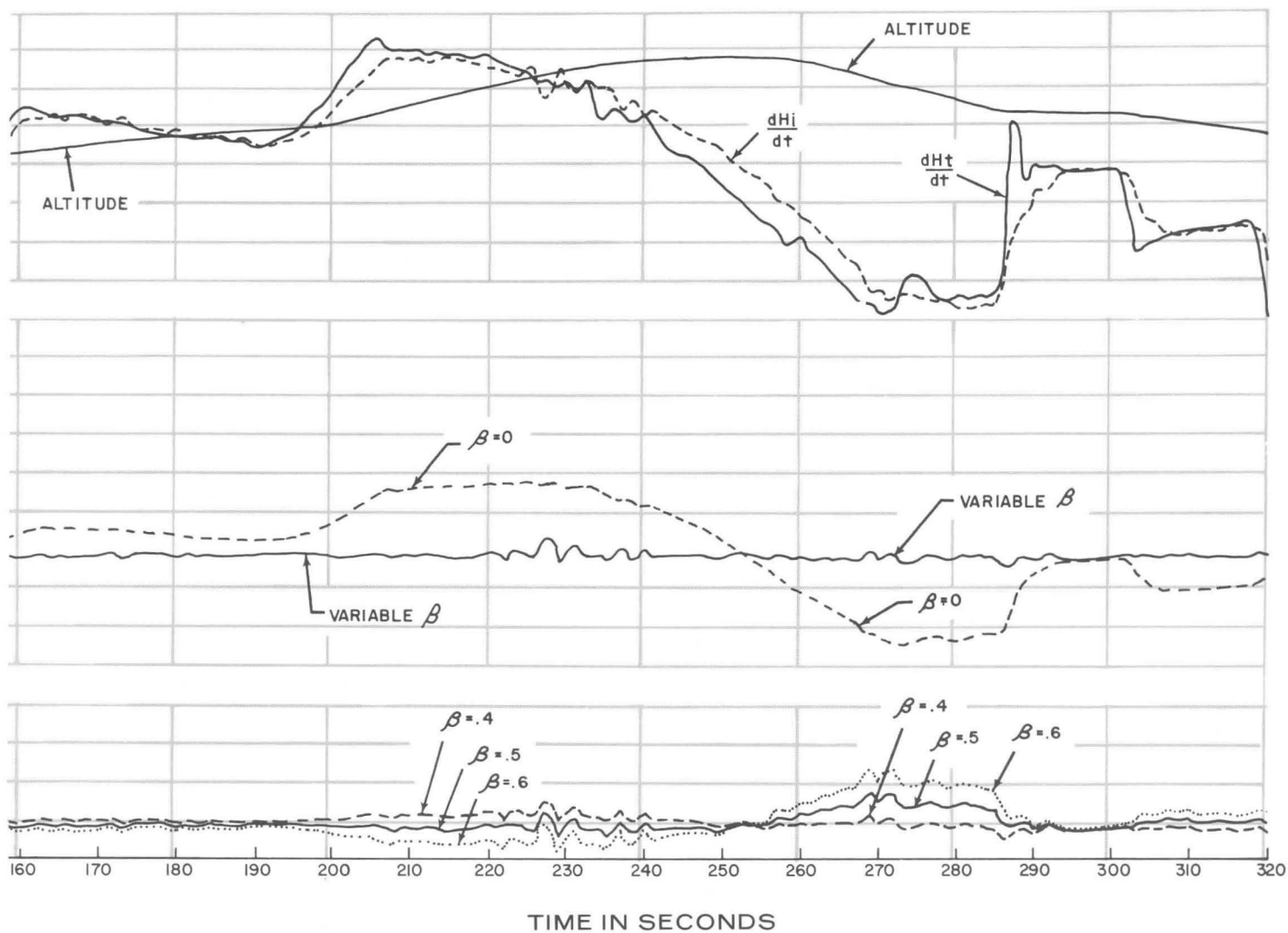
$$E = 100 \frac{H_t - H_{il}}{H_t} (\%) \quad (23)$$

where H_t is the known true pressure altitude and where H_{il} is the indicated altitude, corrected for instrument error and for lag. H_{il} was computed differently for each curve. Inspection of the curves reveals that only when β was variable did the net error remain less than about 0.5 percent. All curves for constant β result in errors well above 0.5 percent during some portion of the "flight."

FIGURE 13

DATA TIME HISTORIES FOR LABORATORY
SIMULATED FLIGHT & RESULTANT ERRORS
FOR VARIOUS LAB CORRECTION METHODS





5 CONCLUSIONS

Accuracy requirements for altitude data in the flight test programs of many of today's aircraft necessitate a correction of indicated altitude for lag in the static system. Methods currently employed for lag correction utilize a constant value of the lag parameter β as predicted by theory. The experiments discussed in this report have shown that, in reality, the lag parameter varies with both altitude and rate of change of altitude. It has further been demonstrated that good lag corrections can be obtained using the theoretical equation, but with a variable lag parameter to be determined by a set of ground lag check experiments on the static pressure system. For reasonable accuracy, β versus H need only be obtained for one or two values of dH/dt .

Instrument system temperature is a parameter included in the lag correction expression; however, for most test conditions errors in estimating this temperature will have a negligible effect on the correction. Experiments established the fact that the surface temperature of the pitot-static probe has no measurable influence on the pressure lag.

Using the methods of lag correction outlined in this report, indicated altitude can be adjusted to true pressure altitude to an accuracy of 0.5 percent. Other instrumentation uncertainties prevent the attainment of greater accuracy.

references

1. Wildhack, W. A., "Pressure Drop in Tubing in Aircraft Instrument Installations," NACA Technical Note 593, February, 1937.

2. De Juhasz, K. J., "Graphical Analysis of Delay of Response in Airspeed Indicators," Jour. Aero. Sci., Vol. 10, No. 3, March, 1943.

3. Huston, W. B., "Accuracy of Airspeed Measurements and Flight Calibration Procedures," NACA Technical Report 919, June 1948.

4. Draper, C. S., and Schliestett, G. V., "General Principles of Instrument Analysis," Instruments, Vol. 12, No. 5, May, 1939.

5. Head, R. M., "Lag Determination of Altitude Systems," Jour. of Aero. Sci., Vol. 12, No. 1, January, 1945.

6. Vaughn, H., "The Response Characteristics of Airplane and Missile Pressure Measuring Systems," Aeronautical Engineering Review, November, 1955.

7. Newman, B. G., "Lag in Airborne Pressure Measuring Systems," National Aeronautical Establishment, Canada, Report No. LR-100, April, 1954.

8. Lamb, J. P. Jr., "The Influence of Geometry Parameters Upon Lag Error in Airborne Pressure Measuring Systems," WADC Technical Report 57-351, ASTIA Doc. No. AD 130 790, July, 1957.

9. Smith, K., "Pressure Lag in Pipes with Special Reference to Aircraft Speed and Height Measurements," Royal Aircraft Establishment Report No. Aero. 2507, November, 1954.

10. Charnley, W. J., "A Note on a Method of Correcting for Lag in Aircraft Pitot-Static Systems," Royal Aeronautical Establishment Report No. Aero 2156, September, 1946.

11. Bartlett, E. P., "Flight Test Handbook; Performance, Part II," Air Force Flight Test Center TN-59-22, ASTIA Doc. No. AD-215865, July, 1959.

12. Kendall, J. M., "Time Lags Due to Compressible-Poiseuille Flow Resistance on Pressure-Measuring Systems," Naval Ordnance Laboratory Memorandum 10677, ASTIA Doc. No. ATI-88875, May, 1950.

13. Sinclair, A. R., and Robins, A. W., "A Method for the Determination of the Time Lag in Pressure Measuring Systems Incorporating Capillaries," NACA Technical Note 2793, September, 1952.

14. Minzner, R. A., and Ripley, W. S., "The ARDC Model Atmosphere, 1956, AFCRC TN-56-204, ASTIA Doc. No. 110233, December, 1956.

APPENDIX I

derivation of pressure lag equation

Consider a long, straight tube of length L , diameter D , and cross sectional area A open at one end and connected at the other end to an enclosure of volume V . As the pressure changes at the open end (station 1), flow will take place and there will be a pressure gradient along the tube resulting in a pressure difference between the open end and the enclosure. The pressure difference, $P_1 - P_2$, is the lag in the system

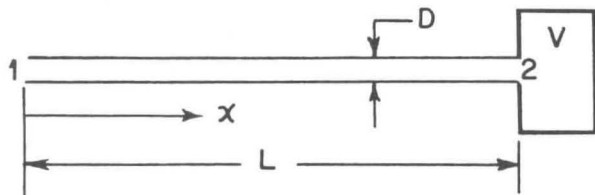


Figure 14

For steady laminar flow along the tube the governing flow equations are the Hagen - Poisseuille law

$$\frac{dP}{dx} = - \frac{32\mu u}{D^2} \quad (24)$$

and the continuity equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho u) = 0 \quad (25)$$

with the boundary condition that at $x = L$

$$A\rho_2 u_2 = V \frac{d\rho_2}{dt} \quad (26)$$

Equation (26) states that the mass flow into or out of the tubing at $x = L$ is equal to the rate of mass change in the volume V .

Experimentally it has been shown that for steady changes of pressure at the source, the rate of pressure change is nearly constant along the tube

$\left[\frac{dP}{dt} \neq f(x) \right]$. Then from the polytropic relationship between pressure and density ($P = \rho^n \text{constant}$), one can show that $d\rho/dt$ will also be essentially independent of x .

Equation (25) may be integrated using equation (26) to evaluate the constant of integration. The resultant expression can be solved for u , obtaining

$$u = \frac{1}{\rho} \frac{d\rho_2}{dt} \left[(L - x) + \frac{V}{A} \right] \quad (27)$$

Substituting this expression for u into equation (24) one obtains

$$\frac{dP}{dx} = - \frac{32\mu}{D^2} \left[(L - x) + \frac{V}{A} \right] \frac{1}{\rho} \frac{d\rho_2}{dt} \quad (28)$$

Since the pressure lag, $P_1 - P_2$, is very small compared to P_1 , the slight variation of ρ along the tube can be neglected when integrating equation (28). Then, setting $\rho = \rho_2$ and integrating, the following expression for the pressure lag is obtained:

$$\Delta P = P_1 - P_2 = \frac{32\mu}{D^2} \left[\frac{L^2}{2} + \frac{VL}{A} \right] \frac{1}{\rho_2} \frac{d\rho_2}{dt} \quad (29)$$

$$\Delta P = \frac{128L\mu}{\pi D^4} \left[\frac{AL}{2} + V \right] \frac{1}{\rho_2} \frac{d\rho_2}{dt} \quad (30)$$

The density terms can be changed to pressure terms using the polytropic relationship.

$$\Delta P = \frac{128L\mu}{\pi D^4} \left[\frac{AL}{2} + V \right] \frac{1}{n P_2} \frac{dP_2}{dt} \quad (31)$$

APPENDIX II

description of a static pressure system lag checker

Numerous methods of measuring static and total pressure lag parameters in aircraft instrument systems have been proposed. The simplest, but least adequate, is the application of a step function pressure change at the pressure source and measurement of the system response time constant. The step function method does not provide a realistic evaluation of the response of the system to continuous, smooth pressure changes which are normally experienced in flight.

Several devices have been built at the Air Force Flight Test Center to accomplish a lag check as described in AFFTC-TN-59-22, the "Flight Test Handbook; Performance Part II." As has been shown in the main text of this report, the method described in the "Flight Test Handbook" is inadequate for many modern aircraft test programs. A system must be devised which will provide data from which to extract the parameter β as a function of altitude and rate of change of altitude.

Primarily a static pressure system lag checker must meet two specifications:

1. Capability to vary rate of change of altitude from 5,000 to 30,000 feet per minute.
2. Capability to maintain the required rates up to at least 60,000 feet.

Figure 15 schematically shows a system which will provide the required lag check information and meet the listed specifications.

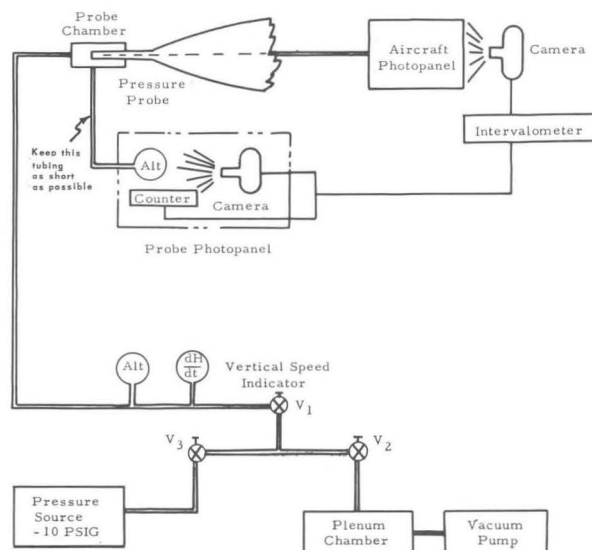


Figure 15
SCHEMATIC OF STATIC PRESSURE SYSTEM LAG CHECKER

A vacuum pump is used to provide simulated rate of climb capability. A plenum chamber is shown in conjunction with the vacuum pump. Its function is to damp out any pulsations of the pump. If the pump provides smooth rates of pressure change, no plenum chamber will be required. The vacuum pump must be capable of sustaining a 30,000 feet per minute rate of climb to 60,000 feet.

A pressure source is necessary for a simulated rate of descent. Available compressed air lines, regulated to about 10 psig (25 psia) will be sufficient. A pressure greater than atmospheric pressure is necessary to maintain a high rate of descent (20,000 to 30,000 feet per minute) all the way from 60,000 to 2,000 feet. Venting to the atmosphere will not provide sufficient pressure differential for high descent rates below a pressure altitude of about 20,000 feet.

Note that by use of valves V_2 and V_3 either pressure or vacuum can be selected. Valve V_1 is used to control the flow rate to maintain the desired rate of change of pressure altitude. If available, a vertical speed (rate of climb) indicator is placed between V_1 and the probe in clear sight of the operator. This allows the operator to hold a fixed pressure altitude change rate by manipulation of V_1 to maintain a selected reading on the vertical speed indicator. The vertical speed instrument must be capable of indicating rates of altitude change up to 30,000 feet per minute. To the author's knowledge the only vertical speed indicator designed to date with such a range is the Model V-3 Vertical Speed Indicator manufactured by Specialities, Inc. of Syosset, Long Island, New York. Other instruments may be available or special units could be designed for this purpose.

If a vertical speed indicator cannot be obtained or built to monitor the specified rates, an altimeter can be used by carefully monitoring the rate of needle rotation. This method is only marginally satisfactory. It is extremely difficult to maintain rates of altitude change close to a specified constant value. The resultant data is generally poor, and the operation is complicated. However, if no other means is available, the method will provide better β versus H data than other methods which result in a constant value for β .

When a vertical speed indicator is used, an altimeter should be placed adjacent to it as shown in Figure 15. The operator monitors this altimeter to keep aware of the approximate system pressure altitude. Without altitude monitoring capability, the operator

could easily over-pressurize the system in a simulated descent.

A small, leak-proof chamber is placed over the pitot-static probe. The total pressure opening in the probe should be left open to avoid damaging differential pressures which could occur in the air-speed or Mach number instruments. The probe enclosure should be connected by a short tube of at least one quarter inch diameter to a small photopanel for recording simulated altitude. The probe photopanel needs only contain one or more altimeters and a frame counter for a static system lag check. Averaging the readings of multiple altimeters will avoid any effects of peculiarities of one individual altimeter. The altimeter should be calibrated at least every 400 feet for both increasing and decreasing altitude. The altimeters in the probe photopanel and in the aircraft photopanel must be set at 29.92.

Pulses to actuate the camera in the probe photopanel should come from the same intervalometer as the pulses to the aircraft photopanel camera. Thus if the counters in both photopanels are initially zeroed, correlation of data between the probe and the aircraft photopanel will be no problem. Time histories of the required parameters can be obtained using the aircraft timer.

In testing static system lag, it is imperative that the pressure at the probe never exceed sea level pressure. An alert operator monitoring the altimeter near valve V_1 can avoid instrument overpressure by closing the control valve as the altimeter nears sea level pressure altitude. If further precaution is desired, a differential pressure transducer can be mounted in such a position that it will sense the difference between the probe chamber pressure and ambient pressure. When the probe chamber pressure becomes greater than ambient pressure the differential pressure transducer can close a solenoid valve installed between the pressure source and valve V_3 , automatically prohibiting further pressure increase.

The system which has been described provides experimental values for static pressure lag as a function of time and altitude. The altitude time histories can readily be differentiated to obtain rate of change of altitude. With that data the parameter β can be found from

$$\beta = \frac{\Delta H}{\frac{P_{SL}}{P_i} \frac{dH_i}{dt}} \quad (32)$$

where ΔH is the difference between the calibrated values of the probe altitude and the indicated aircraft altitude at the same time. The term P_i is the pressure corresponding to H_i for standard day conditions.

As mentioned previously, the specifications given in this report are only valid for lag measurements in the static pressure system. The lag checker could be modified to provide total pressure system lag data by replacing the vertical speed indicator and the altimeter used for monitoring purposes with correspondingly appropriate instruments for the pressure rates and ranges required for the total pressure system. The instruments in the probe photopanel would also have to be changed.

APPENDIX III

lag correction procedure outline

The test engineer desiring to employ the method of lag correction described in this report will find it somewhat difficult using the main text as a procedural guide. For that reason this Appendix is included to outline the steps required in making altimeter lag corrections. The user of this outline is cautioned, however, to familiarize himself with the descriptive information of the text so that errors in interpretation of the outline will not be made and application of the methods beyond their validity will not be attempted.

Determining β Versus H From Ground Lag Check:

The experimental apparatus required for the determination of the lag parameter β is described in Appendix II. For most test programs ground lag data need only be taken for one positive and one negative

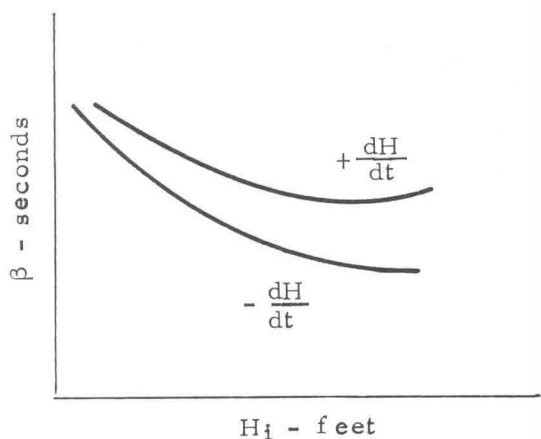
rate of change of altitude. The values for each should be about two-thirds the maximum rate at which flight test data will be acquired. For example, if flight test data is needed between +30,000 feet per minute and -30,000 feet per minute, ground lag checks should be made at $\pm 20,000$ feet per minute. The simulated climbs should begin at an altimeter reading between sea level and 5,000 feet and should extend to 10,000 feet above the maximum altitude to which the aircraft will be flown under test conditions. Similarly, the simulated descents should be initiated at 10,000 feet above the maximum expected altitude. The extra altitude margins are needed to avoid inclusion of transient data in the flight test altitude range.

The data from the lag check system is processed as follows:

1. Correct all altimeter readings for instrument error.
2. $\Delta H = H_1 - H_i$ (feet)
3. $\frac{dH_i}{dt}$ = time rate of change of H_i (feet per second)
This term may be calculated numerically as $\Delta H_i / \Delta t$ or it may be obtained by applying a curve fit to the H_i time history and differentiating that equation. The latter method will minimize the data scatter.
4. Convert H_i to P_i through the Standard Atmosphere relationships for altitude and pressure.
5. Calculate $\frac{P_{SL}}{P_i}$ using consistent units. $P_{SL} = 29.92126$ inches Hg
6. Calculate β from

$$\beta = \frac{\Delta H}{\frac{P_{SL}}{P_i} \frac{dH_i}{dt}} \text{ (seconds)}$$

7. Plot or tabulate β as a function of H_i for both the positive and the negative rates of change of altitude.



Flight Test Data Lag Correction:

The quantity to be determined is H_{il} , indicated altitude which has been instrument corrected and corrected for lag. (H_{il} in this report is the same as H_{icl} in Reference 11.)

The following data is available from ground tests and flight records:

- β vs H_i ; from ground lag check
- T_g = ambient temperature at time of ground lag check
- H_i = indicated altitude (instrument corrected)
- t = time
- T = instrument system temperature (estimated average value)
This is only required if it deviates from T_g by 100 degrees F or more.

The data reduction procedure is as follows:

1. Correct all altimeter readings for instrument error.
2. Obtain β for each value of H_i from plots of β versus H_i . Use $+ dH/dt$ curve for flight data taken during climbs and $-dH/dt$ curve for flight data taken during a descent. When dH_i/dt is zero the correction is zero.
3. Convert H_i to P_i through Standard Atmosphere relationships for altitude and pressure.
4. Calculate $\frac{P_{SL}}{P_i}$ using consistent units.
5. Calculate $\frac{dH_i}{dt}$ either by incremental method ($\frac{\Delta H_i}{\Delta t}$) or by

fitting a curve to the H_i time history and differentiating. dH_i/dt must have units of feet per second.

6. If the instrument system temperature is more than about 100 degrees F different from the ambient temperature at the time of the ground lag check compute the temperature correction, β/β_g , from

$$\frac{\beta}{\beta_g} = \left(\frac{T}{T_g} \right)^{3/2} \left(\frac{T_g + S}{T + S} \right)$$

$S = 198.6$ if T and T_g are degrees Rankine

$S = 110.4$ if T and T_g are degrees Kelvin

7. $\Delta H =$ lag correction =

$$\beta \left(\frac{\beta}{\beta_g} \right) \frac{PSL}{P_i} \frac{dH_i}{dt} \text{ (feet)}$$

If T is within 100 degrees F of T_g use

$$\frac{\beta}{\beta_g} = 1$$

8. $H_{il} = H_i + \Delta H$ (feet)

